The limiting system corresponding to the Boussinesq approximation for small Rossby numbers is reduced to the form (5.2) (see Ref. 6). The system (5.2) may be of interest in connection with the investigation of the formation of sunspots, where magnetic and convective effects are coupled.

The theory of magnetohydrodynamic flow of a heavy fluid at small Alfvén numbers which has been outlined above is similar from the conceptual standpoint to the theory of the flow of a heavy rotating fluid at small Rossby numbers. There is also a great analogy between the static equilibrium approximation (3.6) discussed in Sec. 3 and the classical quasigeostrophic approximation in meteorology [6].

LITERATURE CITED

1. P. Germain, "Introduction à l'étude de l'aéromagnétodynamique," Cahiers Phys., No. 103 (1959).
2. R. Kh. Zeitunyan (Zeytounian), "Notes sur les écoulements rotationnels de fluides parfaits," Lecture Notes in Physics, Vol. 27, Springer-Verlag, Berlin (1974).
3. H. Grad, "Mathematical problems in magnetofluid dynamics and plasma physics," in: Proc. of the Int. Congr. of Mathematicians, Stockholm (1962).
4. R. Kh. Zeitunyan (Zeytounian), "Invariants lagrangiens et intégrales premières en magnétodynamique des fluides," Appl. Sci. Res., 32, No. 6 (1976).
5. A. S. Monin, Weather Forecasting as a Problem in Physics, MIT Press, Cambridge, Mass. (1972).
6. R. Kh. Zeitunyan (Zeytounian), "La météorologia du point de vue du Mécanicien des Fluides," in: Fluid Dynamics Transactions, Vol. 8, Polish Acad. of Sci., Warsaw (1976).

SYMMETRIC COLLISION OF TWO-LAYER JETS OF AN IDEAL INCOMPRESSIBLE LIQUID

S. A. Kinelovskii and Yu. A. Trishin

UDC 532.522

1. We consider the problem of finding the potential flow arising after the symmetric collision of plane two-layer free jets of an ideal incompressible liquid. Assuming that the flow is steady state, we shall analyze the conditions that must be satisfied in this case by the flows in the different layers of the colliding jets. For simplicity, by virtue of the symmetry, we can replace the plane of symmetry with a rigid stationary wall and consider the stationary problem of a two-layer jet of an ideal incompressible liquid hitting this wall. The flow in each of the layers of the jet is characterized by its value of the Bernoulli integral constant. Assuming that the pressure at infinity and on the free streamlines is zero, we denote by h the ratio of the Bernoulli integral constants in the layers;

$$h = \frac{\frac{1}{2}\rho_1 v_1^2}{\frac{1}{2}\rho_2 v_2^2}, \tag{1.1}$$

where v_1 and v_2 are the liquid velocities in the layer at infinity, and the subscripts 1 and 2 are assigned, respectively, to the external (outside the wall) and internal layers of the two-layer jet. In the general case the densities of the layers, ρ_1 and ρ_2, and the velocities, v_1 and v_2, are different. In addition, the problem also depends on the geometric parameters specified at infinity, such as the thicknesses of the layers and the angle of inclination of the jet to the wall. Depending on the values of all these parameters, it is possible in principle to have three variants of the flow arising when a two-layer jet hits the wall; a) The forward jet (the pestle) is inhomogeneous, while the return jet, (the cumulative jet) is homogeneous; b) the pestle and the cumulative jet are homogeneous; c) the pestle is homogeneous, while the cumulative jet is inhomogeneous.

Figure 1 shows the flow configuration corresponding to condition a), with a homogeneous jet and an inhomogeneous pestle, where ρ_1 is the density of the liquid layer external to the wall, ρ_2 is the density of the liquid layer inside the wall, and their velocities at infinity are v_1 and v_2; δ_1 and δ_2 are the thicknesses of the layers of the incident jet at the point at infinity, B; δ_1 is the thickness of the external layer of the pestle at the

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 42-52, March-April, 1980. Original article submitted June 5, 1979.

0021-8944/80/2102-0193 $07.50

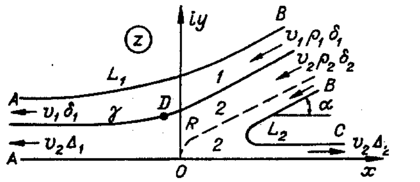

Fig. 1

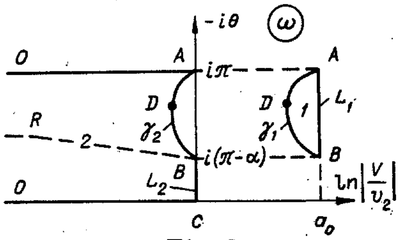

Fig. 2

point at infinity, A; Δ_1 is the thickness of the internal layer, and Δ_2 is the thickness of the cumulative jet at the point C. The angle of inclination of the velocity vector of the layers of the incident jet to the wall at the point B is $\theta_0 = -(\pi - \alpha)$. The region of flow of the external layer is bounded by the free streamline L_1, on which the velocity of the liquid is v_1, and the separation curve between the layers, γ, which is also a streamline. The region of flow of the internal layer of the incident jet is bounded by the streamline γ, the free streamline L_2, on which the velocity of the liquid is v_2, and the abscissa axis, on which the angle of inclination of the velocity vector is equal to zero for $x > 0$ and equal to $-\pi$ for $x < 0$.

Let us consider the regions corresponding to this flow in different complex-variable planes. The regions of flow in the physical plane $z = x + iy$ are shown in Fig. 1. The regions of flow in the plane of the complex potential $w = \varphi + i\psi$ in this case are obvious: For the external layer this will be a strip of width $q_1 = \delta_1 v_1$, and for the internal layer it will be a strip with a slit, of the same kind as in the classical theory of cumulation; the total width of this strip will be $q_2 = \delta_2 v_2 = \Delta_1 v_2 + \Delta_2 v_2$. The regions of flow in the plane of the logarithm of the complex velocity

$$\omega = \ln \frac{dw}{v_2 dz} = \ln \left| \frac{V(z)}{v_2} \right| - i\theta(z),$$

where V and θ are, respectively, the modulus of the velocity vector and the angle of the inclination to the x axis, are shown in Fig. 2 for each of the layers. For the internal layer this will be a half-strip with a discarded piece bounded by a segment of the ordinate axis and the curve γ_2, corresponding to the curve γ separating the layers. The region corresponding to the external layer of the jet is bounded by a vertical line which has the coordinate $a_0 = \ln (v_1/v_2)$ on the abscissa axis and by some curve γ_1 corresponding to the separation curve γ.

Now let us consider the different values of the ratio of the Bernoulli integral constants in the layers of a two-layer jet. If the quantity h defined by (1.1) is equal to unity and $v_1 = v_2$, then it is obvious that both layers consist of the same liquid, there is no discontinuity between the velocities at the curve γ, and we have classical problem of a jet of an ideal incompressible liquid hitting a stationary wall (the problem of the symmetric collision of free jets). If the Bernoulli integral constants in the layers are equal ($h = 1$) but $v_1 \neq v_2$, then the streamline γ is the curve of discontinuity of the flow, but it can be shown, as is done in the classical theory of cumulation, that the flow can be described by continuous analytic functions. Mathematically this means that the solutions for each of the regions 1 and 2 in the complex-potential plane can be analytically continued through the separation curve between these regions from one region into the other (see, e.g., [1]). Geometrically this means that the regions corresponding to the flow in each of the layers in the ω plane (see Fig. 2) can be made to coincide by an additional shift of one of them along the abscissa axis by an amount

$$a_0 = \ln \frac{v_1}{v_2} = \frac{1}{2} \ln \frac{\rho_2}{\rho_1},$$

so that region 1 takes the place of the cut-out portion in region 2, curves γ_1 and γ_2 coincide, and the total region will be a half-strip, as in the classical case. Physically this means that the flows of the liquid in the two layers are dynamically similar and can be obtained from each other by a dynamic similarity transformation. The situation is much more complicated if the Bernoulli integral constants in the layers are not equal, i.e., $h \neq 1$. In this case the solution for one region cannot be analytically continued into the other, and in the ω plane the curvilinear segments γ_1 and γ_2 of the boundaries of regions 1 and 2 will not coincide with each other when region 2 is shifted parallel to the abscissa axis by an amount a_0 (see Fig. 2). The flow will be described, as noted in [2], by piecewise analytic functions. To solve the problem, we must find the solution for each of the regions 1 and 2 in Fig. 1 and sew these solutions together along the streamline γ. The condition for sewing the solutions together is obtained from the condition that the pressures are equal in regions 1 and 2 on the separation curve γ; it can be written in the form

$$\rho_1 \left(v_1^2 - V_1^2(\beta)\right) = \rho_2 \left(v_2^2 - V_2^2(\beta)\right), \tag{1.2}$$

where $V_1(\beta)$ and $V_2(\beta)$ are the moduli of the liquid velocity on the separation curve γ in the corresponding regions, which are parametrically given by the angle of inclination β between the curve γ and the x axis. In terms of functions of a complex variable, condition (1.2) means that the complex potentials of the flows in regions 1 and 2 in Fig. 1 on the separation curve γ between these regions must satisfy the condition

$$\rho_1 v_1^2 - \rho_1 \left| \frac{dw_1}{dz} \right|^2 = \rho_2 v_2^2 - \rho_2 \left| \frac{dw_2}{dz} \right|^2, \quad \arg \frac{dw_1}{dz} = \arg \frac{dw_2}{dz}.$$

To prove the existence of a solution of this problem and construct it in general form is very difficult, since in this case the usual mathematical apparatus developed for solving jet problems proves inadequate. However, from physical considerations, it is logical to assume that since a stationary jet flow with a velocity discontinuity along the separation curve between the layer of the jet exists for h = 1, then a stationary jet flow also exists when the Bernoulli integral constants in the layers differ from each other, at least by a small amount. This assumption was used in [2, 3] to construct the solutions of jet problems with a flow discontinuity at the boundary of adjacent jets with respect to pneumonic problems. As was shown by experimental investigations of jet amplifiers, the effect of the liquid in the zone of collision between the jets is small in comparison with the effect of the pressure gradient, which justifies idealizing the flow and replacing the jet-mixing region developing downstream with the velocity discontinuity curve [3]. Extending some of the reasoning in [3, 4] to our case, we can assume that there exists a unique solution of this problem for values of h in a small neighborhood of the value h = 1.

2. We consider the stationary configuration for a two-layer jet hitting a wall, shown in Fig. 1, and assume that the Bernoulli integral constants in the layers are equal, i.e., h = 1. In this case, as noted above, there exists a unique solution of the problem. Let $V_{10}(\beta)$ and $V_{20}(\beta)$ be the velocities of the liquid in each of the layers on the separation curve γ, where β – the angle of inclination of the velocity vectors on γ to the x axis – is a parameter determining each point of the curve γ, with $-\pi \le \beta \le -(\pi - \alpha)$. Since the Bernoulli integral constants are equal, for the velocities of the layers at the point at infinity and at any point on the separation curve γ we have

$$v_2 = \lambda v_1, \quad V_{20}(\beta) = \lambda V_{10}(\beta), \tag{2.1}$$

where $\lambda = \sqrt{\rho_1/\rho_2}$.

Assume that the thickness of the external layer is much less than the thickness of the internal layer ($\delta_1 \ll \delta_2$). Then it is clear from physical considerations that the velocity of the liquid along the separation curve between the layers will differ very little from v_1 in the external layer and from v_2 in the internal layer:

$$V_{10}(\beta) = v_1(1 - \nu(\beta)), \; V_{20}(\beta) = v_2(1 - \nu(\beta)), \tag{2.2}$$

and it is easily shown, taking account of (2.1), that this difference $\nu(\beta)$ will be the same for the two layers. At the ends of the separation curve $\nu(-\pi) = \nu(-\pi + \alpha) = 0$, and the maximum value of $\nu(\beta)$ is attained at some point D on γ (see Fig. 1), where this curve has its maximum curvature (here the pressure for a given streamline is maximum, and the liquid velocities in the layers are minimum).

Let us make the Bernoulli integral constants in the layers somewhat unequal; specifically, for example, let us increase the velocity of the external layer at the point at infinity by a small amount

$$v_{11} = v_1(1 + \varepsilon_0), \tag{2.3}$$

where we can assume that ε_0 is a small quantity of the same order as max $\nu(\beta)$. Changing the velocity v_1 by a small amount causes a small change in the entire flow. On the changed separation curve γ between flow regions 1 and 2 we can write

$$V_1(\beta) = V_{10}(\beta)(1 + \varepsilon_1(\beta)), \; V_2(\beta) = V_{20}(\beta)(1 + \varepsilon_2(\beta)). \tag{2.4}$$

Since the Bernoulli integral constants for regions 1 and 2 are now unequal, the connection between the velocities of the liquid along the boundary between them will be determined not by Eq. (2.1) but by Eq. (1.2), where we must take account of the fact that the velocity in region 1 at infinity is now v_{11}. Substituting Eqs. (2.3), (2.4) into (1.2) and taking account of (2.1), (2.2), we obtain

$$\varepsilon_1(\beta) \approx \varepsilon_2(\beta) + \varepsilon_0. \tag{2.5}$$

From (2.4), (2.5) we can obtain on the curvilinear segments γ_1 and γ_2 of the boundaries of regions 1 and 2 in the ω plane (see Fig. 2) the relations

$$\ln \frac{V_2(\theta)}{v_2} \approx \ln \frac{V_{20}(\theta)}{v_2} + \varepsilon_2(\theta), \quad \ln \frac{V_1(\theta)}{v_2} \approx \ln \frac{V_{20}(\theta)}{v_2} + \varepsilon_2(\theta) - \ln \lambda + \varepsilon_0. \tag{2.6}$$

It can be seen from (2.6) that, to within small quantities of higher order in relation to ε_0, region 1 in Fig. 2 will fit into the cut-out portion of region 2 if we shift it by an amount $a_1 = (\ln \lambda - \varepsilon_0)$ along the abscissa axis. By a process similar to the one used in [1], it can be shown that in this case, with the indicated accuracy, the solution for region 2 in the complex-potential plane can be analytically continued to region 1. We arrive at similar results if we arrange the variation of the Bernoulli integral constant in the external thin layer not by a variation of the velocity (2.3) but by a small change in the density of the liquid.

Consequently, if the Bernoulli integral constants in the layers of the liquid differ by a small amount and if we assume that the thickness of the external layer is small, to within small quantities of higher order, the solution of the flow of a two-layer jet can be found by the classical methods developed for the solution of jet problems.

In conclusion it should be noted that although our discussion above dealt with flow variant a), everything we have said applies equally well to flow variant c) with homogeneous pestle and inhomogeneous cumulative jet.

3. Now let us give more detailed consideration to flow variant b), i.e., the flow condition with homogeneous pestle and homogeneous cumulative jet. To do this, we shall first write out the conditions that must be satisfied by the parameters of our problem in order to realize a particular flow regime. We consider flow variant a) shown in Fig. 1. From the conditions of conservation of the mass and momentum flows, we obtain an expression for the thickness of the internal layer of the pestle;

$$\Delta_1 = \frac{1+\cos\alpha}{2}\delta_2 - \frac{\lambda^2 v_1^2}{v_2^2}\frac{1-\cos\alpha}{2}\delta_1. \tag{3.1}$$

From this, setting $\Delta_1 > 0$, we obtain the condition for the realization of a flow regime with a homogeneous cumulative jet, which we write in the form

$$\delta_2 > \frac{\lambda^2 v_1^2}{v_2^2}\frac{1-\cos\alpha}{1+\cos\alpha}\delta_1. \tag{3.2}$$

In a similar manner we can obtain the condition for the realization of a flow regime with a homogeneous pestle and an inhomogeneous cumulative jet. This condition will have the form (3.2), in which the inequality sign is reversed. Thus, for a two-layer jet hitting a wall, the condition for the realization of flow with a homogeneous pestle and a cumulative jet, i.e., the realization of flow variant b), is the expression (3.2) with an equality sign. At least, by virtue of its existence conditions, this flow regime is an intermediate one, covering the transition between variants a) and c). From this it formally follows that if we successively vary the values of the parameters determining the problem in such a way that, for example, the inequality (3.2) gradually decreases, becomes an equality, and then becomes an inequality in the opposite direction, we must obtain a set of flows whose configuration gradually passes from flow variant a) through variant b), to variant c). In actuality, if we remain within the confines of the adopted scheme with two flow regions, it is impossible to realize such a transition. As the inequality (3.2) approaches an equality, the separation curve γ comes closer to the branching streamline R (see Fig. 1), on which we find the point O of complete stagnation of the flow in region 2.

We shall assume for the sake of definiteness that the Bernoulli integral constant in the external layer of the incident jet is greater than the constant in the internal layer, i.e., $h > 1$, and from (1.2) we can express the instantaneous velocity $V_1(\beta)$ of the liquid in region 1 on the streamline γ in terms of velocity $V_2(\beta)$ in region 2:

$$V_1(\beta) = \frac{1}{\lambda}\sqrt{v_2^2(h-1) + V_2^2(\beta)}. \tag{3.3}$$

From this it follows that the velocity of the liquid in region 1 on the streamline γ cannot be less than a completely defined value, and specifically, everywhere on the separation curve

$$V_1(\beta) \geqslant \frac{v_2}{\lambda}\sqrt{h-1}. \tag{3.4}$$

On the other hand, when the sign in (3.2) is an equality sign, the separation curve γ must merge with the streamline R, as shown in Fig. 3a, and the liquid in region 1 at the point O of this streamline is slowed down to zero velocity, which, according to (3.4), is possible only when $h = 1$, or, to state the same condition another way, when $\lambda^2 v_1^2 = v_2^2$. Thus, the flow regime shown in Fig. 3a can be realized only in the unique case when the Bernoulli integral constants in the layers are equal, i.e., when the flows in regions 1 and 2 are dynamically similar to each other. Moreover, if we use a line of reasoning qualitatively analogous to the one used by M. A. Lavrent'ev [1], we can show that for a flow scheme with bounded derivatives of the velocity for $h \neq 1$ it is impossible to have a flow configuration in which the separation curve γ approaches the branching streamline R sufficiently closely (see Fig. 1).

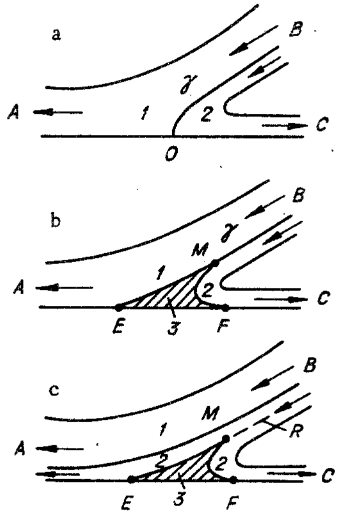

Fig. 3

Let us consider the flow configuration in Fig. 1 and assume that $h < 1$. Then we find that the velocity of the liquid in region 2 on the curve γ satisfies the inequality

$$V_2(\beta) \geqslant v_2\sqrt{1-h}. \tag{3.5}$$

At the same time, by a proper choice of the parameters of the problem we can make sure that the thickness Δ_1 of the internal layer of the pestle is arbitrarily small. In this case the separation curve γ approaches the branching streamline R very closely but does not coincide with it. But on the streamline R, in a neighborhood of the point O, the velocity of the liquid in region 2 is very low, and it is equal to zero at the point O itself, while on the streamline γ the velocity of the liquid is bounded below by condition (3.5), and the condition that the derivatives of the velocity must be bounded is not satisfied.

The question of what flow scheme is realized when the separation curve between the dissimilar layers of a two-layer jet for $h \neq 1$ comes sufficiently close to the branching streamline is one which is of interest in its own right and cannot be answered in advance. It is not impossible that in this case an instability in the flow will develop, and it will be of a nonstationary periodic character. If we attempt to stay within the confines of steady-state flows, then, as a hypothesis, we can propose a flow configuration with a stagnation zone, which is not inconsistent from the hydrodynamic point of view. In Fig. 3b we show such a flow configuration for the case in which the parameters of the problem satisfy Eq. (3.2) with an equality sign, i.e., when the separation curve between the layers coincides with the branching streamline R and we have a flow variant with a homogeneous pestle and a cumulative jet. At some point M, the streamline separating the layers will branch into two streamlines ME and MF, which, together with a segment EF of the wall, bound region 3 of the liquid at rest. On the segment BM of the streamline γ the velocities in regions 1 and 2 are connected with each other by the relation (3.3), while on segments ME and MF of the divided streamline the velocities are constant and are equal to their respective values at the point M. This configuration ensures continuous pressure over the entire flow region. The classical problem of the collision of free jets of an ideal incompressible liquid, with the formation of a stagnation zone (which was considered some time ago by Chisotti), is, as is generally known, insufficiently defined. Depending on the value of the pressure p_0 given in the stagnation zone, it may extend to infinity (if $p_0 = 0$), have finite dimensions (if $0 < p_0 < p_*$), or contract into a point (if p_0 is equal to the pressure of total stagnation p_*). In the case under consideration here, the dimensions of the stagnation zone cannot be less than the values so determined, since the pressure in it must be strictly less than the pressure of total stagnation for the layer with a lower value of the Bernoulli integral constant. Nevertheless, it is impossible to say in advance whether this problem will be sufficiently defined to have a unique solution. In an analogous manner, we can also propose a flow configuration with a stagnation zone for the case when the separation curve between the layers comes sufficiently close to the branching streamline. For flow variant a), when the inequality (3.2) is close to an equality, such a flow configuration is shown in Fig. 3c. The advantage of this configuration over the one shown in Fig. 1 is that it always ensures that we satisfy the boundedness condition for the derivatives of the velocity in the entire region of moving liquid.

Summing up all of the foregoing, we can say that the existence of stationary configurations in all three of the flow variants introduced at the beginning of this study for the symmetric collision of plane two-layer jets of an ideal incompressible liquid, when the Bernoulli integral constants in the layers are not equal to each other,

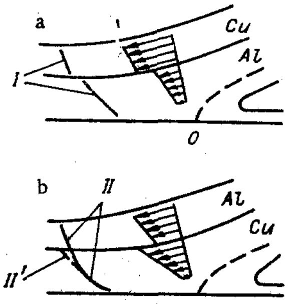

Fig. 4

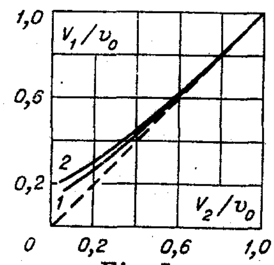

Fig. 5

is not contradictory, and there are a number of factors in support of this assertion (in addition to the qualitative arguments given in this study, these factors also include known experimental results).

4. The authors of [4] investigated experimentally the behavior of dissimilar metals at the interface between them when there is an oblique collision between bimetallic plates. A symmetric scheme for the collision of bimetallic plates was achieved through symmetric throwing of the plates by explosive charges. The existence and direction of a tangential discontinuity in the velocity at the interface between the dissimilar metals was fixed by means of control wires pressed into the bimetallic plates along the normal to the interface.

In their discussion, the authors of [4] arrived at some erroneous conclusions, most of which were based on the thesis that a stationary flow configuration cannot exist when multilayer jets collide, and therefore they were unable to give a sufficiently simple and clear explanation of the fact that the control wire broke. An analysis of these results on the basis of all the factors we have mentioned showed that the experimental results of [4] can be completely explained by means of the ideal-incompressible-liquid model.

The jet model of the collision of bimetallic plates investigated in [4] is characterized by different values of the liquid density in the layers and equal values of their velocity at infinity. The relation (3.3), which gives the connection between the velocities of the liquid in the layers along the streamline γ separating them, will in this case take the form

$$V_1(\beta) = \frac{1}{\lambda} \sqrt{(\lambda^2 - 1) v_0^2 + V_2^2(\beta)}, \tag{4.1}$$

where v_0 is the common velocity of the layers at infinity. This relation was stated in [4], but the authors did not pay enough attention to it. It follows from (4.1) that the heavier liquid at the interface will always (except at the point at infinity) have a higher velocity, which explains the experimental results obtained. It should be borne in mind that these results were obtained for a flow regime with an inhomogeneous pestle and a homogeneous cumulative jet, which is shown in Fig. 1.

In the first series of experiments described in [4], the external layer (copper) was heavier than the internal layer (aluminum). According to the foregoing discussion, on the separation curve between the layers, at any point on it we have $V_1(\beta) > V_2(\beta)$, and consequently when collision occurs, the part of the control wire in the copper must move forward farther than the part of the wire in the aluminum. This fact was recorded in the experiments (Fig. 3 of [4]) and is qualitatively illustrated in Fig. 4a, where the number I indicates the position of the wire in the layers after collision of the bimetallic plates in the first series of experiments.

A qualitatively analogous result should be obtained in the third and fourth series of experiments of [4], where the external layer is Duralumin and the internal layer is aluminum. The conclusion of the authors of [4] that in this case there should be no displacement of the layers relative to each other, because the densities of the layers are practically equal, is incorrect. The fact is that the magnitude of the displacement of the layers is an integral characteristic which depends not only on the difference between the layer densities but also on a number of other factors. Using the example of the pair of metals mentioned, we can examine qualitatively the causes that can bring about a noticeable displacement of the layers even when they have almost equal densities. In [4] the density of the Duralumin used is not indicated, but its approximate value is known. For example, for A12024 alloy the tabulated value of the density is $\rho_1 = 2.785$ g/cm^3, and for A1921T alloy $\rho_1 = 2.833$ g/cm^3. The densities of some other aluminum alloys also lie in this range. For clarity, we shall work with the densities of the two alloys mentioned above. For the density of the internal layer (aluminum) we use the value $\rho_2 = 2.71$ g/cm^3. Then the value of the parameter λ in our problem will be $\lambda = 1.013$, e.g., the external layer has the density of A12024 alloy and $\lambda = 1.022$ for the density of the second alloy. As can be seen, the densities of the layers on a relative scale do in fact differ by a small amount: The parameter λ differs from unity by about 1.5-2%. The connection between the velocities of the layers along the separation curve in the jet model is given by relation (4.1); this relation is shown graphically in Fig. 5. Here the velocities are referred to their values at infinity, curves 1 and 2 relate, respectively, to the smaller and larger values of λ, and the dashed straight line is the line of equal velocities. Any curve constructed on the basis of (4.1) is the diagram of the possible values of the velocities in each of the layers along the separation curve γ. Indeed, moving along the separation curve γ from the point at infinity B (see Fig. 1), in Fig. 5 we will go down from the point (1, 1) along the curve of possible states. The descent will continue until we reach the point D on γ, at which this streamline has maximum curvature. As we move further along γ to the point at infinity A on the $V_1(V_2)$ curve, we shall ascend from the point of minimum velocity values in the opposite direction. On the $V_1(V_2)$ curve the velocity minimum point will be lower as the point D corresponding to it on γ is closer to the stagnation point O (see Fig. 1). On the other hand, the jump in velocities from one layer to the other at this point will be maximum on γ, which is obvious from Fig. 5. Consequently the first factor that can cause a substantial difference between the velocities of the layers (and a noticeable displacement of the layers) for a small difference between their densities is the geometry of the flow. For example, for a relative velocity of 0.2 in the internal layer, the velocity in the external layer at this point of the curve, in the case of the Duralumin-aluminum pair considered here, will be 50% higher. If the parameters of the collision are such that the separation curve comes sufficiently close to the point of complete stagnation of the liquid, then the difference between the velocities of the layers may be sufficiently large to bring about a noticeable displacement of the layers. Another factor is the length of time the velocity difference acts. If the difference between the velocities of the layers remains relatively small everywhere on γ but exists for a sufficiently long time, then the final displacement of the layers may be substantial. In the case of an ideal liquid the difference between the velocities on γ in our case exists everywhere except at the point at infinity, and consequently it exists for an infinitely long time, but an ideal liquid is only a model of a real process. For metals this model is valid at sufficiently high pressures, when the structural-strength forces can be disregarded. The region in which high pressures are attained when there is a collision between the plates is of limited length, and it is the dimension of this region that determines in our case the real time of action of the velocity difference at the interface between the layers. As a practical matter, this time depends on the geometric and dynamic parameters of the collision and on the structural-strength properties of the plate materials. Thus, the mere fact that the difference between the layer densities in a bimetallic plate is small does not mean that there will in fact be no displacement of the layers or that the displacement will be very small. The experiments in [4] confirmed these conclusions. The direction of the displacement of the control wire in the specimens of the third and fourth series of experiments (see Fig. 5 of [4]) is analogous to that in the first series (see Fig. 4a) and indicates that the heavier layer has a higher velocity.

Now let us consider the results of the second series of experiments in [4]. The results of this series include a fact which was not explained in [4]. Specifically, while in the other series of experiments the control wire on the boundary between the layers broke and the two parts of the wire which were in the different layers moved some distance apart, in this series there was no break of the wire in the specimens. The position of the control wire in the specimen for one of the experiments of this series in [4] was shown on a photograph (see Fig. 4 of [4]) and is qualitatively illustrated in Fig. 4b of our study, where the number II indicates the control wire. According to all of the foregoing discussion, the part of the wire in the heavier internal layer should pass the wire in the external layer, and the configuration of the control wire in the specimen should be qualitatively the same as the one indicated by the number II' in Fig. 4b. However, there is no significant disagreement between our reasoning and the results of these experiments. Let us consider some facts which relate to this problem. In the first place, this series of experiments differs from the first series in having the positions of the external and internal layers of the bimetallic plates interchanged, while the thicknesses of the layers remained unchanged. However, this exchange substantially alters the geometric characteristics of the flow. Using the data of the table in [4], which gives the values found for the main parameters determining the collision of the plates, and using formula (3.1), taking account of the fact that $v_1 = v_2$, we can calculate the thickness of the internal layer of the pestle (see Fig. 1) for the first and second series of experiments. It turns out that in the first series, where the internal layer is aluminum, $\Delta_1 = 1.23$ mm, and in the second series, where the internal layer is copper, $\Delta_1 = 1.85$ mm, i.e., in the first case the curve of separation between the layers was substantially closer to the point of total stagnation of the liquid (in comparison with the initial thickness of the internal layer, $\delta_2 = 2$ mm). This means, as was shown above, that the discontinuity in the velocities along the separation curve between the layers, which makes the internal layer lag behind in the first series, is considerably greater than the velocity jump on γ, which must have made the external layer lag behind in the second series. Another important factor arising out of the real properties of the materials is the effect of viscosity. Let us consider some point on the separation curve between the layers and, in a direction normal to the separation curve, construct diagrams of the liquid velocities in the two layers. If this point is not very close to the point of total stagnation of the flow and the neighborhood considered is not too large, we may assume as an approximation that on a segment perpendicular to the streamline γ all the liquid velocity vectors are perpendicular to this segment. For the case in which the external layer is denser (the first series of experiments), the velocity diagram at an arbitrary point of the separation curve between the layers is shown qualitatively in Fig. 4a, and for the case in which the internal layer is heavier (second series) it is shown in Fig. 4b. An analysis of Fig. 4 shows that the presence of the real viscosity of the materials, manifested in friction between adjacent elementary layers (streamlines) in each layer of an inhomogeneous jet, will affect the flow differently in the two cases under consideration. In the first case (see Fig. 4a) the diagram of velocities is such that the viscosity will cause a displacement of the layers relative to each other, and conversely, in the second case (see Fig. 4b) the real viscosity will prevent any displacement of the dissimilar layers with respect to each other. These two facts are enough to explain why in the first series of experiments in [4] there was a break in the control wires of the specimens, while in the second series, in which the positions of the metals were interchanged, there was no break. It is beyond doubt that for the case in which the internal layer is heavier than the external layer, we can select such materials for the layers of the bimetallic plates and obtain such collision parameters that the internal layer will nevertheless move "ahead" of the external layer and the configuration of the control wire in the specimen will be qualitatively close to the configuration indicated in Fig. 4b by II in the external layer and II' in the internal layer.

It should be emphasized that the phenomenon of a discontinuity in velocities on the interface between the dissimilar metals is determined by the actual structure of the flow corresponding to the investigated collision of the plates, not to the structural strength and viscosity of the metals, as is asserted in [4]. The real properties of the materials must be taken into consideration only in order to gain an understanding of the fact that such characteristics of the materials as strength or viscosity may intensify or weaken the result produced by this phenomenon, which in this case is expressed in a displacement of the layers with respect to each other and a break in the control wire.

Thus, all the experimental results of [4] are completely reconcilable with the stationary model considered in the present study for the collision of two-layer jets of an ideal incompressible liquid with different values of the Bernoulli integral constant in the different layers of the jet, and therefore the results of [4] qualitatively confirm the validity of our model.

LITERATURE CITED

1. M. A. Lavrent'ev and B. V. Shabat, Problems of Hydrodynamics and Their Mathematical Models [in Russian], Nauka, Moscow (1973).
2. P. M. Belotserkovskii, "A nonlinear problem concerning the collision of two-dimensional jets of an ideal incompressible liquid with a flow discontinuity on the boundary between the jets," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1970).
3. P. M. Belotserkovskii, "Collision of two jets issuing from channels with parallel walls for different velocities at the free surfaces," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1969).
4. N. S. Kozin, V. I. Mali, and M. V. Rubtsov, "Tangential explosion with collapse of a bimetallic casing," Fiz. Goreniya Vzryva, No. 4 (1977).